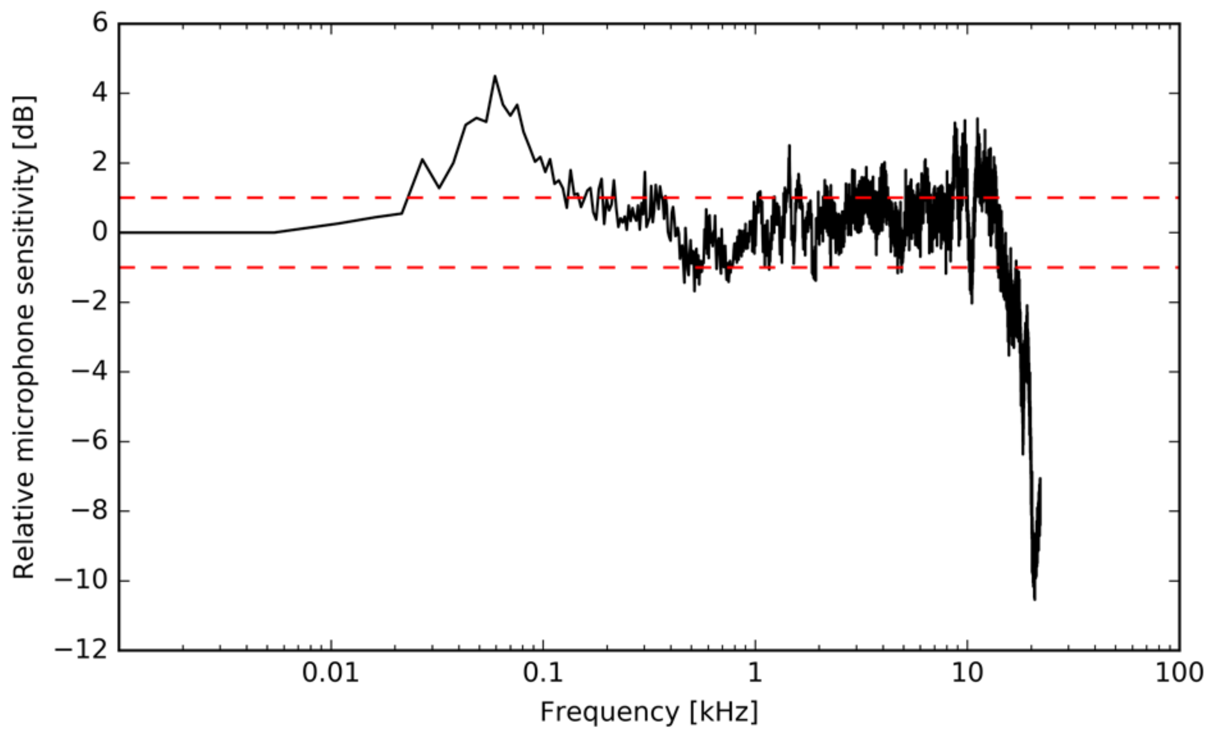
Online Supporting Material for

ACOUSTIC MODAL TESTING OF BICYCLE RIMS

Matthew Ford, Patrick Peng, and Oluwaseyi Balogun

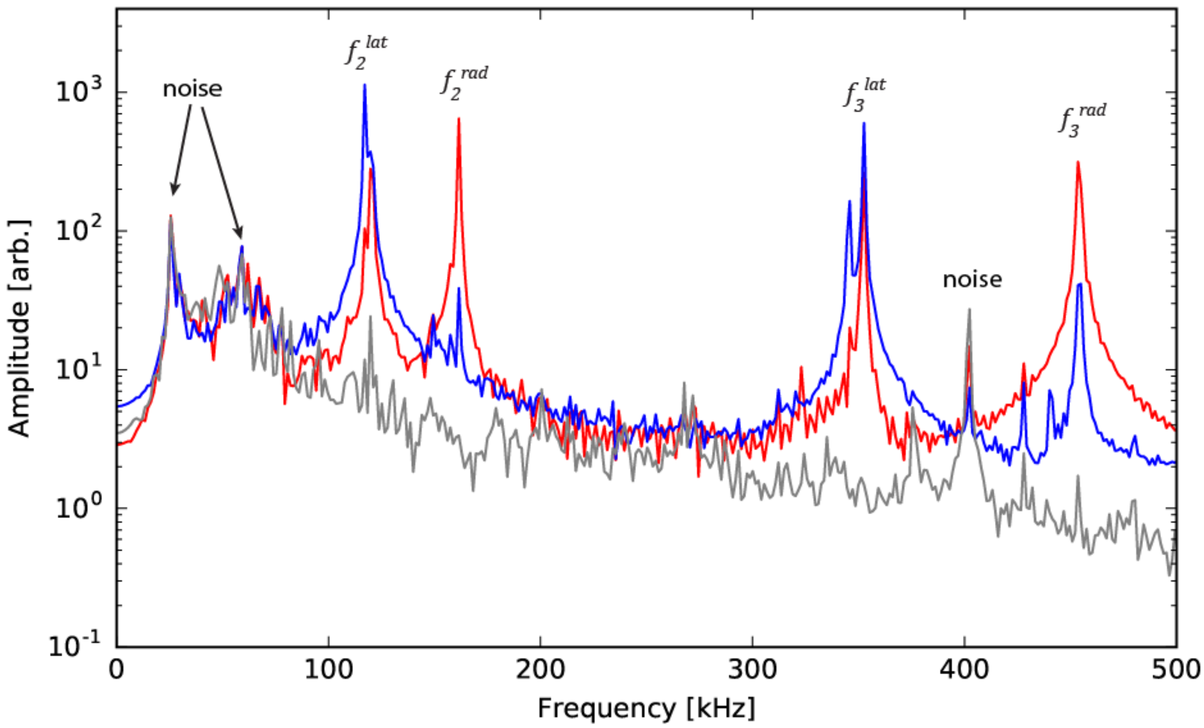
METHOD: FREQUENCY RESPONSE OF SMARTPHONE MICROPHONE

The frequency response of the built-in microphone in an Apple iPhone SE (model A1662) was measured in an anechoic chamber. The smartphone and a calibrated reference microphone with a flat frequency response (Etymotic Research ER-7C Probe Mic System) were placed on a foam block 66 inches from a single mono speaker (Roland MA-12C Micro Monitor). Approximated pink noise was generated from an online source (https://mynoise.net/NoiseMachines/whiteNoiseGenerator.php) and played through the speaker. A spectral average was obtained from both microphones using the Fast Fourier Transform with a buffer size of 8192 samples at 44.1 kHz sample rate with 50 averaging windows and discarding the phase. The iPhone microphone relative sensitivity was calculated by taking the ratio of the iPhone spectrum to the reference spectrum and normalizing by the amplitude at 5.38 Hz.



PEAK IDENTIFICATION PROCEDURE

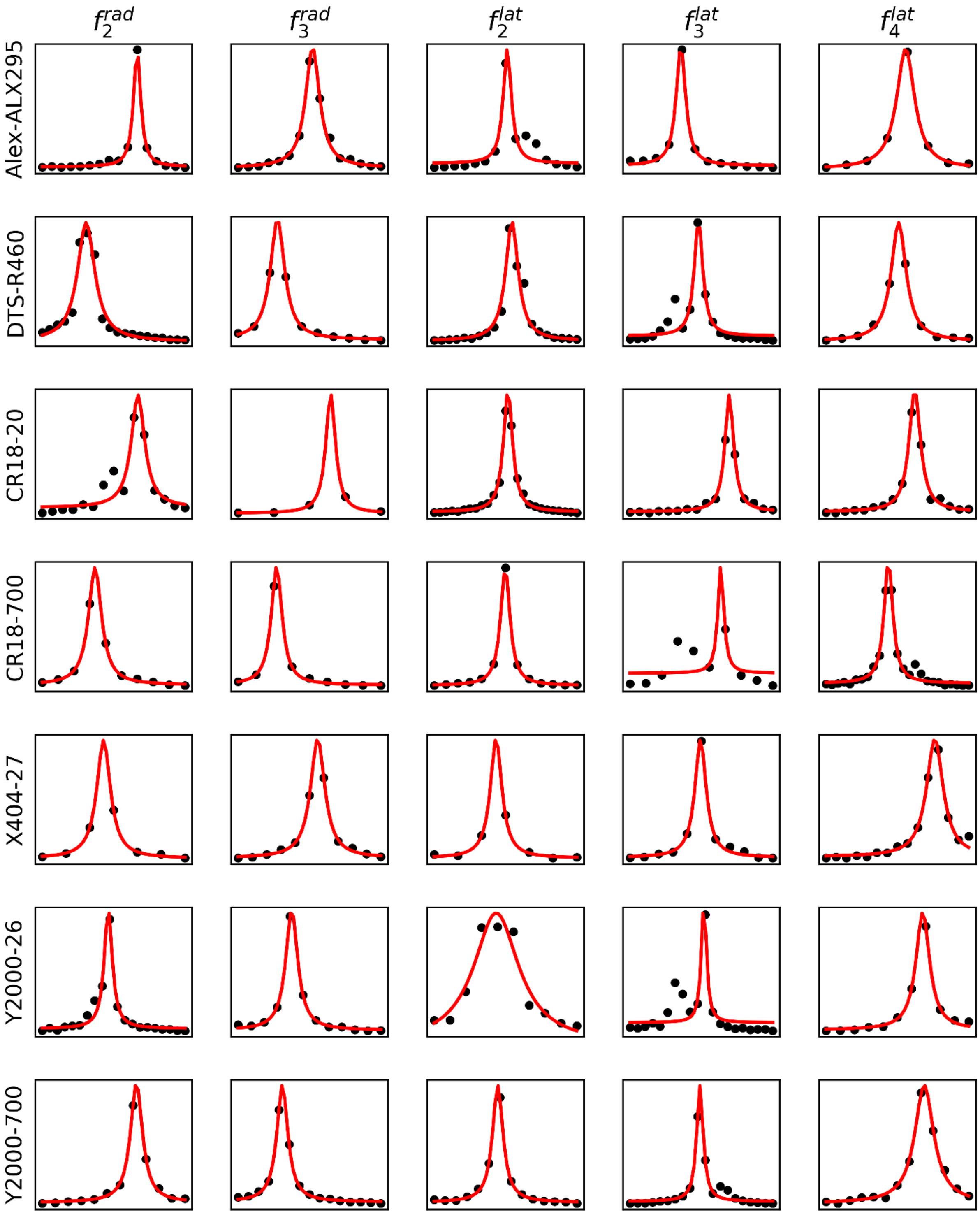
The following procedure was used to identify the radial and lateral mode frequencies for each rim: First, the two spectra were compared with the noise spectrum to identify any peaks with a signal-to-noise ratio of at least 10 (note the first two apparent peaks at 27 Hz and 60 Hz are both present in the noise spectrum and can therefore be discarded). Next, the lowest peaks were compared between the lateral and radial spectra to find duplicates. In the case of duplicates, the peak was assigned to the spectrum with the greater relative magnitude. An illustration of the peak identification procedure is illustrated below:



After identifying the approximate location of each peak, the precise peak parameters were determined by fitting a Lorentzian peak function of the form:

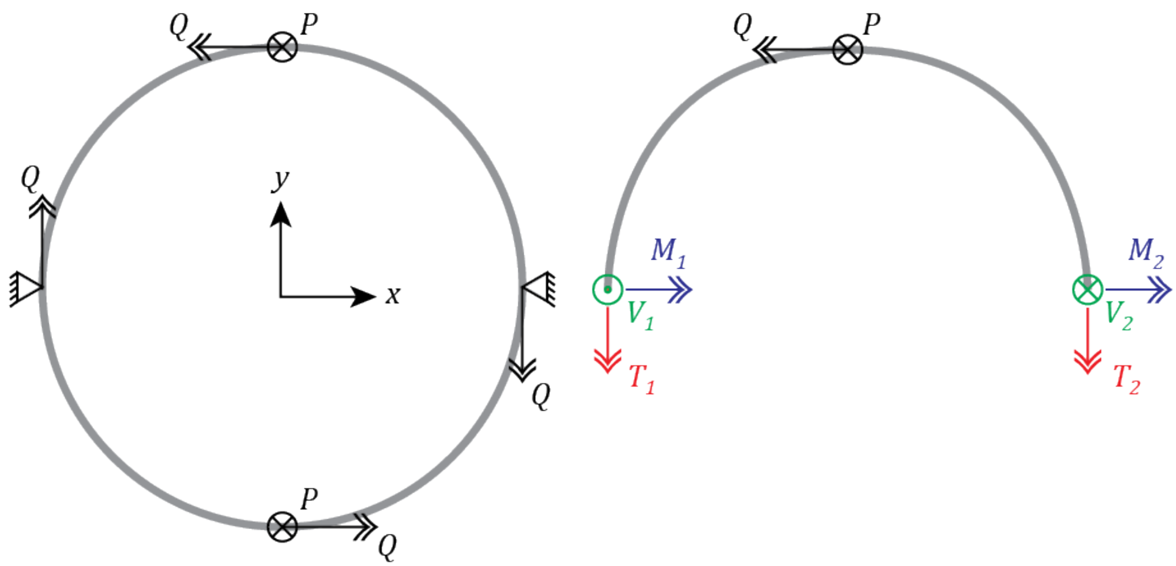
$$F(f) = \frac{1}{2\pi} \frac{\Gamma}{(f - f_0)^2 + (\Gamma)^2}$$

Fitted curves for $f_2^{rad}, f_3^{rad}, f_2^{lat}, f_3^{lat}, f_4^{lat}$ are shown below:


f_2^{rad}
f_3^{rad}
f_2^{lat}
f_3^{lat}
f_4^{lat}
Alex-ALX295
DTS-R460
CR18-20
CR18-700
X404-27
Y2000-26
Y2000-700

ANALYSIS OF THE FOUR-POINT BENDING TEST

In the four-point bending test, the rim is supported at 3- and 9-o'clock and loaded at 12- and 6-o'clock with an out-of-plane force P. A "dummy torque" Q is applied at each point in the same sense as the rotation of the cross-section. Free-body diagrams of the complete rim and upper section are shown below:



The symmetry of the problem gives us the condition

$$M_1 = M_2$$

Sum of forces in the z-direction gives

$$V_1 = \frac{P}{2}$$

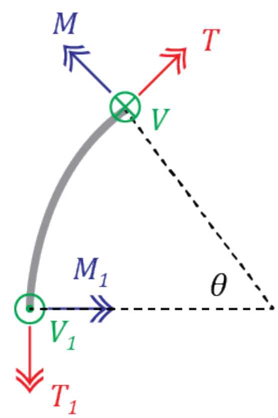
Sum of moments at the right end about the y-axis gives

$$T_1 = T_2 = 0$$

Sum of moments at the right end about the x-axis gives

$$M_1 = \frac{PR}{2} + \frac{Q}{2}$$

The internal forces can now be determined by making a cut at an arbitrary location θ, as shown below:



Sum of forces in the z-direction gives

$$V = \frac{P}{2}$$

Sum of moments at the left end about the x-axis gives

$$M_1 + T\sin\theta - M\cos\theta - \frac{PR}{2}\sin\theta = 0$$

Sum of moments at the left end about the y-axis gives

$$T\cos\theta + M\sin\theta + \frac{PR}{2}(1-\cos\theta) = 0$$

Solving for T and M gives

$$T = -PR(\sin\theta + \cos\theta) + \frac{PR}{2} - \frac{Q}{2}\sin\theta$$

$$M = PR(\cos\theta - \sin\theta) + \frac{Q}{2}\cos\theta$$

The strain energy in the upper half of the rim is given by

$$U = 2\int_0^{\frac{\pi}{2}} \left(\frac{M^2}{2EI_{22}} + \frac{T^2}{2GJ}\right) R\, d\theta$$

The displacement and rotation at the load point is determined using Castigliano's theorem

$$u_0 = \frac{\partial U}{\partial P}, \quad \phi_0 = \frac{\partial U}{\partial Q}$$

This is the "balanced" deflection, i.e. the vertical deflections at each load point assuming that the slope $du/d\theta$ is zero at the supports. In the un-balanced four-point bending test (three points are constrained and the third is loaded), the displacement will be $2u_0$.

$$u_l = -\frac{PR^3}{2GJ}[2(3-\pi) + \mu(2-\pi)] \quad \phi_s = -\frac{PR^2}{8GJ}(1+\mu)(2-\pi)$$

ERROR ESTIMATION FOR STIFFNESS PARAMETERS

All error estimates are made assuming that the relative uncertainty on the rim radius R and mass M are both 1%. The uncertainty on all frequencies are given by $U_{f_n} = \max[\sigma_{f_n}, \Delta f]$, where σ_{f_n} is the square root of the variance of the estimated frequency parameter calculated from the Lorentzian fit, and Δf is the frequency resolution of the spectral average (1.35 Hz).

For radial bending stiffness, the following approximation is used for the propagated uncertainty.

$$EI_{11} = g(R, M, f_2^{rad}) = 2\pi R^3 M \left(\frac{5}{36}\right) \left(f_2^{rad}\right)^2$$

$$U_{EI_{11}} = \sqrt{\left(\frac{\partial g}{\partial R}\right)^2 U_R^2 + \left(\frac{\partial g}{\partial M}\right)^2 U_M^2 + \left(\frac{\partial g}{\partial f_2^{rad}}\right)^2 U_{f_2^{rad}}^2}$$

For the lateral-torsional stiffness, the uncertainty on the stiffness ratio μ were first calculated by the Monte-Carlo method. 1 million random samples for both f_2^{lat} and f_3^{lat} were drawn from normal distributions with a mean equal to the frequency and standard deviation equal to the frequency uncertainty. The uncertainty on μ is then

$$U_\mu = \text{stdev}\left[\frac{16 - \left(f_3^{lat}/f_2^{lat}\right)^2}{9\left(f_3^{lat}/f_2^{lat}\right)^2 - 64}\right]$$

$$GJ = h(R, M, \mu, f_2^{lat}) = 2\pi R^3 M \left(\frac{4\mu + 1}{36}\right) \left(f_2^{lat}\right)^2$$

$$U_{GJ} = \sqrt{\left(\frac{\partial h}{\partial R}\right)^2 U_R^2 + \left(\frac{\partial h}{\partial M}\right)^2 U_M^2 + \left(\frac{\partial h}{\partial f_n^{lat}}\right)^2 U_{f_2^{lat}}^2 + \left(\frac{\partial h}{\partial \mu}\right)^2 U_\mu^2}$$